

DTU



Proof Assistants and Related Tools

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Introduction

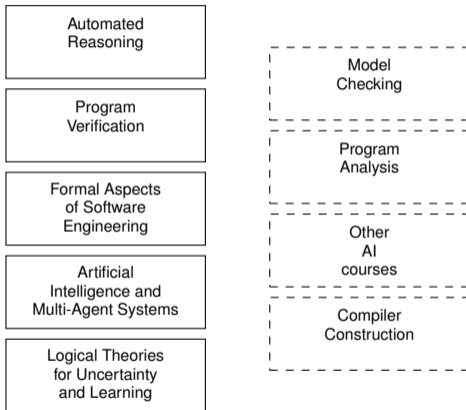
- *Technical* University of Denmark – undergraduate software *engineering* students
- Undergraduate logic course: Logical Systems and Logic Programming (80+ students)
- Graduate logic course: Automated Reasoning (40+ students)
- Undergraduate course leads into many other courses and potentially a thesis project
- 50% lectures and 50% supervised exercise sessions, plus homework assignments
- Danish custom: almost all students pursue a MSc degree

The surrounding curriculum

Year				
1	2	3	4	...
BSc			MSc	
Discrete Mathematics (mandatory)	Functional Programming (mandatory)	Logical Systems and Logic Programming	Automated Reasoning	
Introductory Programming (mandatory)	Computer Science Modelling (mandatory)		Program Verification	
Algorithms and Data Structures 1 (mandatory)	Algorithms and Data Structures 2		Formal Aspects of Software Engineering	
	Introduction to Artificial Intelligence		Artificial Intelligence and Multi-Agent Systems	
	Introduction to Machine Learning and Data Mining		Logical Theories for Uncertainty and Learning	

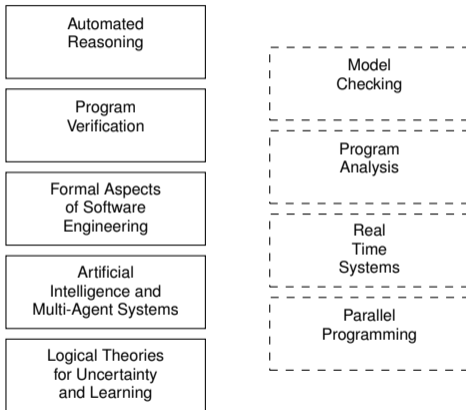
Axioms and inference rules

- Why pick one over the other?
- Notation is widely used
- Special case: type systems



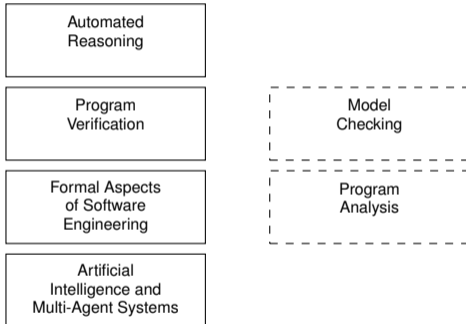
Extensions to a logic

- In practice, first-order logic is not enough
- Examples:
 - Modal Logic
 - Separation Logic
 - Epistemic Logic
 - Hoare Logic
 - Higher-order logic



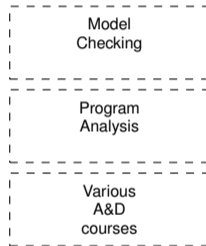
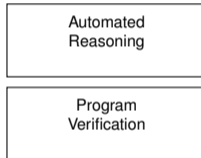
Soundness and completeness

- Basic understanding is enough
- Sound approximations
- Proof systems



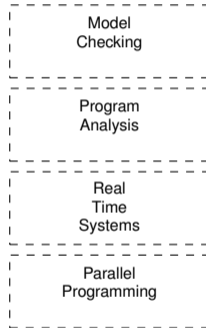
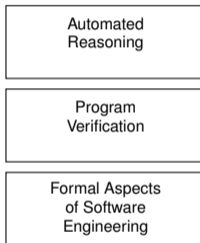
Decidability

- Basic understanding is enough
- Techniques for proving termination
- Only practical examples are useful!



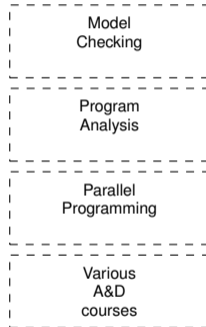
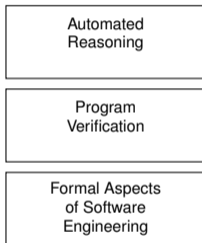
Writing formal specifications

- Verification tools need formal specifications
- Connection to the real world
- Approximations and models



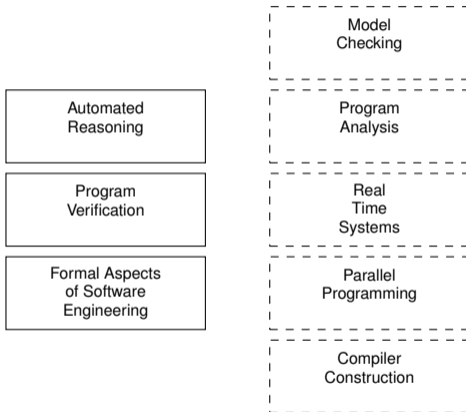
Writing formal proofs

- Needed for all proofs when they become difficult
- Many techniques to explore and practice
- Focus should be on strategies, not particular proofs!



Proof algorithms

- How do automated theorem provers work?
- The foundation of formal analysis tools
- Special case: type checkers



Representing knowledge

- Foundation of artificial intelligence
- Logic programming
- Databases

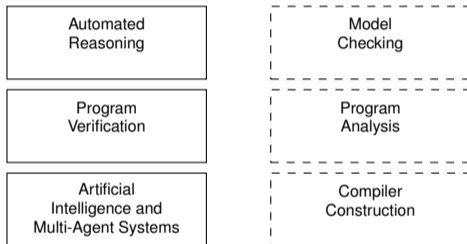
Artificial
Intelligence and
Multi-Agent Systems

Logical Theories
for Uncertainty
and Learning

Other
AI
courses

Implementations

- Theory is not enough
- Complexity of binders, substitution, etc. is often swept under the rug
- Often much harder than expected – see e.g. the POPLmark challenge



Topics I did not mention

- “Advanced” set theory
 - Belongs in a math course
- Recursion theory
 - Non-computable functions are not useful for computer science undergraduates
 - Complexity theory belongs in courses on algorithms and data structures
- Non-finite model theory
 - Quantifier elimination and decidability belongs in a course on automated reasoning
- Non-structural proof theory
 - Proof-theoretic semantics are interesting, but belongs in a specialized course

Summary of topics

- Axioms and inference rules
- Extensions to a logic
- Soundness and completeness
- Decidability
- Writing formal specifications
- Writing formal proofs
- Proof algorithms
- Representing knowledge
- Implementations

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Difficult to practice with pen and paper

Proof assistants

- What are they good for?

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- All the things!
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- And more:
 - Confidence when experimenting
 - Instant feedback on definitions and proofs
 - Implemented examples of proof algorithms and formal proofs