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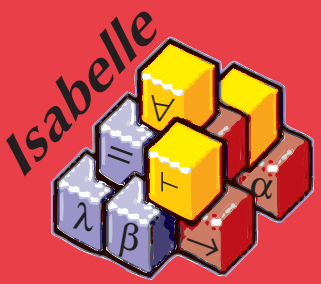

Proofs and Programs Club @ RHUL

Isabelle – an introduction

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What are we doing?

Objectives

- Explain the main differences between Coq and Isabelle
- Write simple proofs in Isabelle/HOL
- Use automation and extraction to verify real programs in Isabelle

Why?

- There's more to the world than constructive mathematics
- Isabelle has some unique advantages
- Jobs: Apple, Intel, IBM, Huawei, Galois, Two Six, Cryspen, ...

Some history

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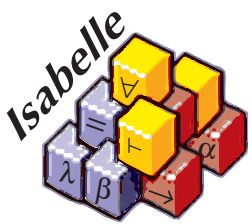
1984	Calculus of Constructions (Coq)
1986	Isabelle

Isabelle



- Generic proof assistant
- Isabelle/HOL is the main logic today
- But also: Isabelle/ZF, Isabelle/Cube, . . .

Editors

- Isabelle/jEdit is the main interface
- Recently, Isabelle/VSCode has become usable

(Isabelle Demo)

HOL vs. CoC

Higher-order logic	Calculus of Constructions
Classical	Constructive
×	Universe hierarchy
Axiom of choice	×
Sets	Functions
All types are inhabited	Inhabitation is evidence

Isabelle vs. Coq

Points for Isabelle

- Sets
- Mixed recursion and corecursion
- Write readable proofs
- Proof by contradiction
- Termination proofs

Points for Coq

- Dependent types
- Subtypes
- Extraction

The Archive of Formal Proofs

Contents

- Extended standard library for things like:
  - Matrices
  - Graphs
  - Instruction set architectures
  - Datastructures
- Proofs of “big” theorems like:
  - Gödel’s incompleteness theorems
  - The undecidability of the continuum hypothesis
  - Fisher’s Inequality
  - Dirichlet’s theorem
  - Lindemann-Weierstraß theorem
  - Green’s theorem

Further reading

Resources on Isabelle: `https://isabelle.systems`

Archive of Formal Proofs: `https://isa-afp.org`

My website: `https://people.compute.dtu.dk/fkjac`

Status on mechanizations of math: `https://www.cs.ru.nl/~freek/100/`

Feel free to email me!

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